STUDY OF CARDIOVASCULAR DYNAMICS WITH RECURSIVE SIMULATOR GENERATION APPROACH

Biswarup Neogi[1], Soumya Ghosal[2], Soumyajit Mukherjee[3], Sinchan Ghosh[4] & Achintya Das[5]

The uninterrupted cyclic activity of the cardiovascular system is initiated by the natural pacemaker i.e. sino-atrium (SA) node situated at the right atrium of cardiovascular system. The movement of the cardiovascular muscle wall due to the inherent force generated in the system may be well understood by the analytical approach of transfer function using a model of the cardiovascular system. The efficacy of this work lies in the measure of the movement of cardiovascular factors in the system. This transfer function analysis is utilized in the analysis of the simulation technique. The simulation is performed with efficient conversion of analog transfer function to sampled data control system using bilinear transformation. The response in sample data domain (S.D) is represented by recursive relation. The recursive system software development approach is presented with the appropriate algorithm.

Keywords: Cardiovascular System Transfer Function, Recursive Relation in Control System, s to z Domain Conversation, Sample Data Domain

1. Introduction

Human cardiovascular system health and its abnormities are so important for clinical researches. Because by knowing the influences of diseases on cardiovascular system properties such as pressure graph the prevention from serious health problems would be possible. To analyze cardiovascular system and effects of diseases on it different ways are usable such as biomechanical model. The heart has four one way valves.

The right atrium receives blood form the body and transfers into the right ventricle, which pumps it into the lungs to receive oxygen. Oxidized bloods then returns from the lungs to the left atrium that in turn propels it into the left ventricle. The left ventricle pumps it around the body for another cycle [1]-[2]. The movement of blood is effected by the residual presser of the blood into the chambers of heart. The construction of heart entails wall motion of the chambers of heart.

[1]Sr.Lecturer, Electronics & Communication Dept. DIATM, Durgapur, India,

[2]Department of Information Technology, RCC Institute of Information Technology, Kolkata, India

[3]Department of Computer Science, Saroj Mohan Institute of Technology, Guptipara, India

[4]Department of Electronics & Communication Engineering, DIATM, Durgapur, India

[5]Department of Electronics & Communication Engineering, KGEC, Kalyani, India

Email: [1]biswarupneogi@gmail.com, [2]soumyaghosal.2008@gmail.com, [3]soumyajitmukherjee.cs@gmail.com, [4]gsinchan@gmail.com

2. Mathematical Model of Cardiovascular System

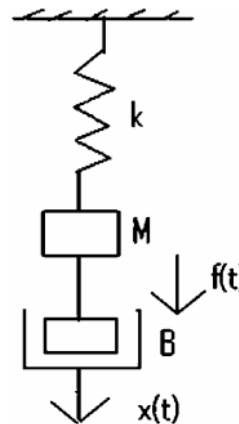The dynamics of the cardiac muscle any be represented by the following model (Fig.1).

Fig.1: Tress Cardiovascular System [1]-[2]

So from the above figure [1]-[2], the dynamical equation for the movement of cardiac muscle may be obtained as:

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} | kx(t) \ = \ f(t) \qquad (1)$$

Where M is the mass of cardiac muscle B is the constant of proportionate for viscous drag and k the constant of proportionate of torsional drag of myocardial cell. c Taking the Laplace transformation on both sides of equation (1), we get,

$$Ms^2X(s) + BsX(s) + kX(s) = F(s) \qquad (2)$$

(Here we have neglected the initial conditions)

where $X(s) = L[x(t)]$ and $F(s) = L[f(t)]$ Here we have taken $x(t)$ as the movement of the cardiac wall for a specimen having mass 'M' which suffers from viscous damping and tensional drag. Now, viscous damping isproportional to the wall movement of the specimen, so that the contribution to this viscous damping may be represented by the expression $B\dfrac{dx(t)}{dt}$.

B' being the constant of proportionality. Tensional drag is proportion to the displacement of the specimen, so that its contribution is given by the expression $kx(t)$, k being the constant of proportionality. $f(t)$ is the force exerted by the cardiac muscle specimen, generated by electrical and electrochemical activity effects on the cardiovascular system. So from equation (2) we can write

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

$$\text{or, } \frac{X(s)}{F(s)} = \frac{K_1}{s^2 + K_2s + K3} = T(s) \qquad (3)$$

Where $K_1 = 1/M$; $K_2 = B/M$; $K_3 = k/M$

Since the cardiac muscle specimen suffers the displacement $x(t)$ due to the input force $f(t)$, the expression in equation (3) represents the transfer function of the cardiac muscle movement due to the force exerted by the cardiac muscle. The cardiovascular wall makes its movement incurring displacement despite hindrance due to frictional and torsional effects of the cardiovascular wall cells. The input force causing the displacement of cardiac muscle is generated by the electrical and electrochemical activity in the cells. The expression of the above transfer function thus gives the measure of the cardiovascular muscle cell wall due to he inherent force generated in the cardiovascular system. The values of the constants K_1, K_2, K_3 in the expression (3) of the transfer function can be determined by experimentation on a cardiovascular system, and the expression (3) thus may be standardized. Thus for known forcing function, the nature of the displacement (movement) of cardiovascular muscle, as function of time can be well predicted taking the Laplace inverse transformation on that equation.

3. PARAMETER DETERMINATION WITH SIMULATOR

From the equation 3, the transfer function of the cardiovascular movement may be written

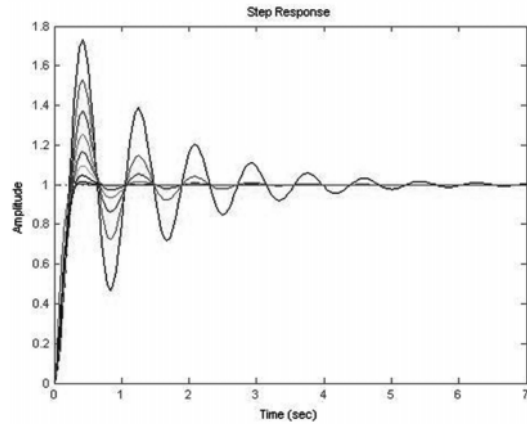$$T(s) = \frac{X(s)}{F(s)} = \frac{K_1}{s^2 + K_2s + K_3}$$

Fig. 2: Step Responses with Different Sets of ζ and ω_n Values

This modeled transfer function response is considered using simulator. The trail and error method is applied in this present situation. Now we compare the expression with the general form of the second order transfer function

$$T(s) = \frac{X(s)}{F(s)} = \frac{\omega_n^2}{s_2 + 2\zeta\omega_ns + \omega_n^2} \qquad (4)$$

The human heart is beating at an average rate of 72 beats per minute [11]-[12]. The frequency of heart beat is f heart $= 72/60 = 1.2$ Hz. The cardiovascular system is a under damped system [1]-[2]. Now,

$$f_d = f_n(1 - \zeta^2)^{1/2} \qquad (5)$$

f_d = damping frequency fn = natural frequency

$$1.2 = f_n(1 - \zeta^2)^{1/2}$$

Dew to various condition of mankind like blood pressure, blood sugar, mental stress, tumor (malignant or benign) etc the hart bit may change causing the variation in the values of damping coefficient. The simulation has been made over different ζ values ranging from 0.1 to 0.9 as shown in the Table 1 and generate the step response (Fig.2) for different natural frequencies. As heart is under damped palpitating organ we can assume that the (damping ratio) $\zeta = 0.9$ for our next study.

Table 1
Damping Coefficient Variation

S E T	Assumed values of Damping coefficient( )	Natural frequencies($_n$)	S E T	Assumed values of Damping coefficient( )	Natural frequencies($_n$)	S E T	Assumed values of Damping coefficient( )	Natural frequencies($_n$)
1	0.1	7.57	4	0.4	8.24	7	0.7	10.5
2	0.2	7.73	5	0.5	8.66	8	0.8	12.5
3	0.3	7.83	6	0.6	9.37	9	0.9	17.5

$$T(s) = \frac{X(s)}{F(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

So the transfer function is

$$T(s) = \frac{X(s)}{F(s)} = \frac{304.218}{s^2 + 31.39s + 304.218} \tag{6}$$

4. THE HEART TRANSFER FUNCTION CONVERSION IN Z DOMAIN

The transfer function originate for representation of human heart input output characteristic should be convert in z domain for the development of artificial heart characteristic.

$$T(s) = \frac{X(s)}{F(s)} = \frac{304.218}{s^2 + 31.39s + 304.218}$$

$$= \frac{304.218s^{-2}}{1 + 31.39s^{-1} + 304.218s^{-2}} \tag{7}$$

Putting the value of S^{-1}, S^{-2} From the s to z conversation table [7]-[9]-[13] we get the expression

$$T(z) = \frac{304.218(T^2/12)\dfrac{(1+10z^{-1}+z^{-2})}{(1-z^{-1})^2}}{1 + 31.39T/2\dfrac{(1+z^{-1})}{1-z^{-1}} + 304.218(T^2/12)\dfrac{(1+10z^{-2}+z^{-2})}{(1-z^{-1})^2}} \tag{8}$$

For simplification let

$$(1 - z^{-1}) = Y, \quad T/2 = X$$

Then

$$T(z) = \frac{304.218X^2/3\dfrac{(1+10z^{-1}+z^{-2})}{Y^2}}{1 + 31.39 \times (1+z^{-1}) + 304.218\ X^2/3\ (1+10z^{-2}+z^{-2})} \tag{9}$$

$$T(z) = \frac{304.218\ X^2(1+10z^{-1}+z^{-2})}{3Y^2 + 94.17XY(1+z^{-1}) + 304.218X^2\ (1+10z^{-1}+z^{-2})} \tag{10}$$

$$T(z) =$$

$$\frac{304.218(X^2 + 10X^2Z^{-1} + X^2Z^{-2})}{3Y^2 + (94.17XY + 94.17XY\ Z^{-1}) + (304.218\ X^2 + 3042.18X^2z^{-1} + 304.218\ X^2z^{-2})} \tag{11}$$

$$T(z) =$$

$$\frac{304.218\ (X^2 + 10X^2z^{-1} + X^2z^{-2})}{(3Y^2 + 94.17\ XY + 304.218\ X^2) + (94.17\ XY + 3042.18X^2)z^{-1} + (304.218\ X^2)z^{-2}} \tag{12}$$

$$T(z) =$$

$$\frac{304.218\ (X^2 + 10X^2z^{-1} + X^2z^{-2})}{\{3Y^2 + 94.17X(1-z^{-1}) + 304.218X^2\} + \{94.17X(1-z^{-1}) + 3042.18X^2\}Z^{-1} + (304.218X^2)z^{-2}} \tag{13}$$

$$T(z) =$$

$$\frac{304.218\ (X^2 + 10X^2z^{-1} + X^2z^{-2})}{3Y^2 + 94.17.94.17Xz^{-1} + 304.218X^2 + \{94.17X - 94.17z^{-1} + 3042.18X^2\}Z^{-1} + (304.218X^2)Z^{-2}} \tag{14}$$

$$T(z) =$$

$$\frac{304.218 + X^2 + 3042.18X^2z^{-1} + 304.218X^2z^{-2}}{(3 - 6z^{-1} + 3z^{-2}) + (94.17X + 304.218X^2 + 3042.18X^2)z^{-1} + (304.218X^2 - 94.17X)z^{-2}} \tag{15}$$

$$T(z) =$$

$$\frac{304.218\ X^2 + 3042.18X^2z^{-1} + 304.218X^2z^{-2}}{3 + (94.17x + 304.218\ X^2 + 3042.18X^2 - 6)z^{-1} + (304.218x^2 - 94.17x + 3)z^{-2}} \tag{16}$$

Taking $S_1 = 304.218\ X^2$, $S_2 = 3042.18\ X^2$, $S_3 = 304.218\ X^2$, $S_4 = 3$, $S_5 = 94.17X + 304.218\ X^2 + 3042.18\ X^2 - 6$, $S_6 = 304.218\ X^2 - 94.17X + 3$.

$$T(z) = \frac{C(z)}{R(z)} = \frac{S_1 + S_2z^{-1} + S_3z^{-2}}{S_4 + S_5Z^{-1} + S_6z^{-2}} \tag{17}$$

Then,

$$C(z)\ \{S_4 + S_5z^{-1} + S_6z^{-2}\} = R(z)\ \{S_1 + S_2z^{-1} + S_3z^{-2}\}$$

$$C(z)\ S_4 + C(z)\ S_5z^{-1} + C(z)\ S_6z^{-2} = R(z)S_1 + R(z)\ S_2z^{-1} + R(z)\ S_3z^{-2} \tag{18}$$

$$C(z) =$$

$$-C(z-1)\frac{S_5}{S_4} - C(z-2)\frac{S_6}{S_4} + R(z)\frac{S_1}{S_4} + R(z-1)\frac{S_2}{S_4} + R(z-2)\frac{S_3}{S_4} \tag{19}$$

Let, $-\dfrac{S_5}{S_4} = G_1, \dfrac{-S_6}{S_4} = G_2, +\dfrac{S_1}{S_4} = G_3, \dfrac{S_2}{S_4} = G_4, \dfrac{S_3}{S_4} = G_5$

Then

$$C(z) = G_1\ C(z-1) + G_2C(z-2) + G_3\ R(z) + G_4R(z-1) + G_5\ R(z-2) \tag{20}$$

Substituting z by k for sampled data control

$$C(z) = G_1\ C(k-1) + G_2C(k-2) + G_3\ R(k) + G_4R(k-1) + G_5\ R(k-2)$$

$$C(0) = G_1\ C(-1) + G_2\ C(-2) + G_3\ R(0) + R(-1) + G_5\ R(-2) = G_3 \tag{21}$$

$$C(1) = G_1\ C(0) + G_2\ C(-1) + G_3\ R(1) + G_4R(0) + G_5\ R(-1) = G_1\ G_3 + G_3 + G_4 \tag{22}$$

$$C(2) = G_1\ C(1) + G_2\ C(0) + G_3\ R(2) + G_4\ R(1) + G_5\ R(0) = G_1\{G_1\ G_3 + G_3 + G_4\} + G_2G_3 + G_3 + G_4 + G_5 \tag{23}$$

$$C(3) = G_1\ C(2) + G_2\ C(0) + G_3\ R(2) + G_4\ R(1) + G_5R(0) = G_1\ [G_1\ \{G_1\ G_3 + G_3 + G_4\} + G_2G_3 + G_3 + G_4 + G_5] + G_2\ G_3 + G_3 + G_4 + G_5 \tag{24}$$

$C(4) = G_1 C(3) + G_2 C(2) + G_3 R(4) + G_4 R(3) + G_5 R(2) = G_1 [G_1 [G_1 \{G_1 G_3 + G_3 + G_4\} + G_2 G_3 + G_3 + G_4 + G_5] + G_2 G_3 + G_3 + G_4 + G_5] + G_2 [G_1\{G_1 G_3 + G_3 + G_4\} + G_2 G_3 + G_3 + G_4 + G_5] + G_3 + G_4 + G_5$ (25)

$C(5) = \ldots\ldots\ldots$

$C(6) = \ldots\ldots\ldots$

$C(7) = \ldots\ldots\ldots$

$:$

$C(N) = \ldots\ldots\ldots$

Applying this recursive method, the output response in sampled data (S.D) control C(K) is determined here. [7]-[8]-[13].

5. THE RECURSIVE SIMULATOR GENERATION APPROACH WITH THE ANCI-C PLATFORM

The program to determine the sample data is very complicated for the severable time recursive action. So the time complexity and the space complexity is much more for this program. In this approach the main program calls the subroutine {Ck} for this recursion.

A. The Algorithm and Supportive Coding Expression for Main Program Algorithm

STEP 1:Input the data of sample periods k and sampling time T, STEP 2: Initial the counter variable (i=1),STEP 3: Ripite the function call Ck(i,T) up to counter variable less or equal to k value(id"k),STEP 4: Print the value of Ck function for each call from STEP3,STEP 5: End the program if counter variable reach the k value.

B. The Algorithm and Supportive Coding Expression for Subroutine Program

The subroutine is produced here for recursive call of output Ck function. Also another the input Rk function is assigned as a unit step in this program.

Algorithm: STEP 1: If k value less than equal to zero then zero assign to function (Ck) will return the value zero else calculate the value of Ck function ie.

$C(k, T) = G_1 C(k - 1) + G_2 C(k - 2) + G_3 R(k) + G_4 R(k - 1) + G_5 R(k - 2)$

$C(k, T) = G_1$ Call Function $C_k(k - 1) + G_2$ Call Function Ck(k – 2) + G3 Call Function Rk(k) + G_4 Call Function Rk (k – 1) + G5 Call Function Rk(k – 2).

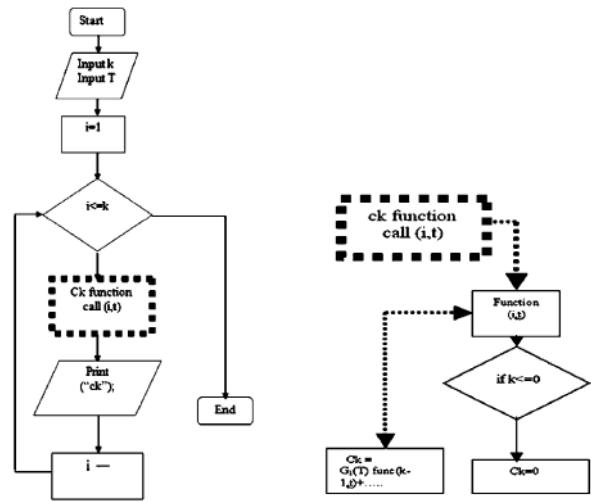## 6. FLOW CHART REPRESENTATION OF RECURSIVE SIMULATOR

Fig. 3: The Main Flow Chart Including the Subroutine Flow Chart of Recursive Approach [13]

7. CONCLUSION

This paper emphasizes the versatile viewed for the displacement of cardiac muscle. The transfer function generation and sample data control action for cardiac system is very effectively important in case of cardiac study. Introducing the recursive software approach it produces some simulative action of heart transfer function. In future the software is in the way of development. Hart is one of the sophisticated nonlinear organ of a human being which controls the flow of the blood through out the body. Recursive simulator will be helpful for acting the nonlinear study of cardiovascular dynamics.

REFERENCES

[1]   Sri. Biswarup Neogi, Dr. Achintya Das "Mathematical Modeling and Simulating of Cardiovascular System Transfer Function with Deterministic Approach of Heart Rhythm Signal Equation", International Conference on Systemics, Cybernetics and Informatics, 2008 (ICSCI-2008) Vol.1of 2 Page-604.

[2]   Biswas, Das, Guha, "Mathematical Model of Cardiovascular System by Transfer Function Method," Calcutta Medical Journal, 103, No.4, 2006 pp.15-17.

[3]   Bo yeon Kim,Yun Seok Chang and Kwang Suk Park, "A Nonlinear Model for Predicting ECG R-R Interval Variation Based on the Evolutionary Computation Approach", Computational Science—ICCS 2003:International Conference, Melbourne, pp 378-386.

[4]   Dividson's Principles and Practice of Medicines,19th Edition. pp 363-364.

[5]   Harrison's Principles of Internal Medicine, 15th Edition, 1, pp 1310.

[6]   William F. Ganong "Review of Medical Physiology", 14th Edition, 1989, pp 53-54.

[7]   Sri. Biswarup Neogi, Dr. Achintya Das, "Recursive Method Introduced in Prosthetic Control Transfer Function", International Conference on Systemics, Cybernetics and Informatics, 2008 (ICSCI-2008), Vol 1of 2, Page-346.

[8]   Dr.Achintya Das & Biswarup Neogi, "Prosthetic Control System of Limb using Artificial Intelligence(AI)", By Published Paper in International Conference ICSCI-2007.

[9]   Achintya Das, "On Electronics Control of Arterial Pressure of Living Body Employing Bio Feedback Techniqu", Phd Awarded Work, pp.103.

[10] Firoozabadi, "Simulating of Human Cardiovascular System and Blood Vessel Obstruction Using Lumped Method" Proceedings of World Academy of Science, Engineering and Technology, 31, July 2008, pp 367-375.

[11] Structure and Function of the Body, 9th Ed, 1992: 231.

[12] www.hypertextbook.com

[13] B.Neogi, S. Mandal, S. Banerjee, A.Das and D.N Tibarewala, "Design of Prosthetic Arms Control Transfer Function with the Analysysis in Digital Domain Inroducing Simulation Aspect", International J. of Engg. Research & Indu. Appls. (IJERIA), 3, No. I (2010), pp 261-274.